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Poverty, Altruism, and Economic Growth

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By examining the role of parental altruism in human capital formation, this paper presents a new mechanism linking fertility and growth. The main conclusion is that parental altruism operates only at higher income levels. Parents start investing in child quality only when their income reaches a certain level. Then fertility rate begins to decline and capital per worker to increase. Thus, income may grow in a non-monotonic pattern. This stratified parental altruism suggests the possibility of a poverty trap. Poor families may converge to a steady state with low income, high fertility, and low human capital investment. Public policies such as subsidy to education and public provision of human capital are viable options for overcoming poverty. Income subsidy policies are shown to be ineffective.

Keywords : overlapping generation model, multiple equilibria, human capital investment, parental altruism, fertility, economic growth, poverty trap.

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I. Introduction

This paper presents a new mechanism linking fertility and growth. The model is built on recent studies that emphasize relations between fertility, human capital accumulation, and growth.[1] It shares common features with these studies; human capital and fertility choices are governed by utility maximization and determine the long-run performance of an economy. However, the model differs from the existing literature primarily in its emphasis on the interactions between poverty and parental choice on human capital.

This paper argues that parental altruism is stratified by income level. Parents enjoy having children and consuming their own income. They also care about their children's future income and seek to improve their quality. A negative correlation between fertility and income results from parent's optimal decision on the quantity and quality of children. However, parents' choice on investment in the quality of children is affected by their income levels. Poor parents are unwilling to spend their income to raise the quality of their children if they are better off without human capital investment. Rich parents always invest a portion of income in their children's human capital. The model establishes that

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1) Based on Barro and Becker(1988, 1989), Becker, Murphy, and Tamura(1990) examines a model in which high human capital raises returns from human capital investments and induces substitution of quality for quantity. Ehrlich and Lui(1991) shows that the implicit intergenerational insurance contract of 'invest and support' may lead to demographic transition. Galor and Weil(1996) shows that if capital is more complementary with women's labor input than men's, increases in capital per worker raise the opportunity cost of children and thus reduce the fertility rate. Dahan and Tsiddon(1998) develops a model in which individuals decide their own education levels on the basis of the wage gap between skilled and unskilled labor. Fertility declines as some poor decide to be skilled.

altruism of parents toward children manifests itself only once the income of parents exceeds a certain threshold level.

The model is designed to encompass some stylized facts on fertility and growth.[2] First, it provides an explanation of non-monotonicity of growth patterns. An economy begins to grow quickly when parents begin go spend a portion of their income to enhance the quality of their children. The human capital of children contributed by parents increases the wages of the children. Furthermore, parents begin to have fewer children when they start investing in quality. Lower fertility raises the level of physical capital per worker, which in turn raises wages of the next generation. When parents' altruism is released by escape form poverty, these two combined effects make wages grow more rapidly. Second, the model shows the existence of a poverty trap. The poor families with low initial income may converge to entrapment in high fertility and little human capital. The poverty trap leads to a persistent income gap among families.

The policy implications of this paper are radical. In the model an income subsidy program is shown to be totally ineffective in overcoming poverty. On the contrary, an income subsidy leads poor parents to have more children, which exactly offsets the increased capital stock of the economy resulting from increased savings. The consequence of the program is that the poor families earn an income as low as before, but have more children. In contrast, this paper shows that a subsidy to education and public provision of human capital can be viable policy options to get escape from poverty.

The remainder of the paper is organized as follows. Section 2 presents the basic model and its dynamics. Section 3 analyzes the effects of several public

2) Recent research on the interaction on fertility and growth has been carried out to explain growth facts observed in the evolution of entire human history (Kremer 1993, Galor and Weil 1999, Hansen and Prescott 1998, Jones 1999, Lucas 1999). While this study, with minor modification, can be considered as one of those models, it is rather narrowly tailored to explain growth facts of a much shorter time span.

policies on income level and its evolution. Section 4 concludes with some discussion on the implications of the model.

Ⅱ. The Structure of the Model

1. Production of Output

We begin by describing a production technology. In every period the economy produces a single homogeneous good using physical capital K and skilled labor H. Output Y is produced according to a constant returns to scale, Cobb-Dougles function:

$$Y_t = AK_t^{\alpha} H_t^{1-\alpha} \tag{1}$$

where A is constant productivity parameter, and $\alpha \in (0,1)$.

Let L denote the total amount of labor used in production, and let h denote human capital of each worker. We assume that the amount of skilled labor is generated according to

$$H_t = L_t h_t \tag{2}$$

The human capital of each worker in this economy is determined by parents' choice on the quantity and quality of children, as will be discussed below.

We can rewrite the production function in per worker terms as

$$y_t = Ak_t^{\alpha} h_t^{1-\alpha} \tag{3}$$

where $k_t = K_t / L_t$. All factors of production are assumed to be paid their marginal products. Then the wage of a unit of labor is

$$w_t = (1-\alpha)Ak_t^{\alpha}h_t^{1-\alpha} \tag{4}$$

Given the structure of production technology, wage is determined by the per worker capital ratio, k_t, and the level of human capital, h_t.

2. Parents′ Preferences

As in Galor and Weil(1997), we consider an overlapping generation model in which people live for three periods. In the first period of their life, people are children and they consume their parents' time and money. In the second period, people supply their labor and earn wages in the market. They choose the number of children to raise and the amount of money to spend for the quality of their children. For convenience, it is assumed that they do not consume in this period. In the last period of their life, people do not work, and they consume their interest-accrued income from savings in the second period. There are no inheritances and there is little opportunity for borrowing against children's future earnings to finance the expenditure on children's human capital investment.

Parents receive utility from the number of children they have (η_t), from the human capital of each child (h_{t+1}), and from the consumption in the last period of their lives(c_{t+1}). Parents' preferences are represented by the utility function

$$U_t = \gamma \ln \eta_t + \delta \ln h_{t+1} + (1-\gamma-\delta)\ln c_{t+1} \tag{5}$$

where $\gamma > 0$ and $\delta > 0$.[3] Note that since the basic unit of counting in this model is the couple, η_t is in fact the number of couples that parents have as their children.

Parents purchase for their children quality goods such as education, health, and other amenities from the market. An individual's human capital, h_{t+1}, is determined by the amount of quality goods consumed as a child. Specifically, if parents purchase q_t units of quality the human capital of the child is

$$h_{t+1}(q_t) = (1+\psi q_t)^{\theta} \qquad \theta \in (0,1) \tag{6}$$

where ψ is a transformation factor that is assumed to be same for all children.

According to (6), quality goods raise human capital, subject to diminishing returns; $h'(q) > 0$ and $h''(q) < 0$.[4] It also shows that if $q_t = 0$, then $h_{t+1} = 1$. If parents do not purchase any quality for their children, their children become unskilled labor, $H_t = L_t$. Thus the production and utility functions include the possibility of zero investment in child quality.

Parents choose the utility maximizing mixture of the quantity and quality of children subject to their intertemporal budget constraints. Parents' full income in the second period is w_t when they do not have any child. Since child rearing requires intensive parental time, there are opportunity costs of having children. Let ρ be the fraction of the time endowment of parents that must be spent in order to raise one child. Then the opportunity cost of having a child is ρw_t and the disposable income of parents with n_t children becomes $(1-\rho n_t)w_t$. The quality goods are purchased by parents from the market. The price of one unit of quality is assumed to be κ, so that providing a child with an quality level q_t costs parents κq_t.

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3) Using other specifications of the utility function does not alter the qualitative nature of this paper's results.

4) This implies that all factors of the production function have monotonically decreasing marginal products. Other work in this area assumes that there are increasing returns to human capital over some range of the production function. See Becker, Murphy, and Tamura(1990).

The full income of parents w_t is divided among expenditure on child rearing, child quality investment, and saving for consumption. Hence, parents in the second period face the budget constraint:

$$\rho w_t n_t + \kappa q_t n_t + s_t \leq w_t \tag{7}$$

where s_t is the amount of saving. In the next period, parents simply consume their saving with accrued interest

$$c_{t+1} = s_t(1+r) \tag{8}$$

where r is interest rate. We assume that interest rate is constant over time.[5)]

3. Optimization

Parents may choose not to invest in child quality and let their children remain unskilled. The optimization problem of this type of parents is to choose the number of children and to save their remaining disposable income, $(1-\rho n_t)w_t$, for consumption. If parents decide to invest in child quality, their children work with a certain level of skill, $h_{t+1} > 1$. These parents choose the optimal mixture of the number of children, the quality of children, and their own consumption, so as to maximize the intertemporal utility. Therefore, it is necessary to analyze the optimization problem corresponding to the discrete choices on child's human capital formation.

Maximization of (5) with respect to n_t and q_t subject to the constraint (6), (7), and (8) yields the number of children parents want to have

5) Empirical evidence indicates that the real rate of return to capital exhibits no upward or downward trends. See Barro and Sala-i-Martin(1995).

$$\eta_t = \begin{cases} \dfrac{\gamma}{(1-\delta)\rho} & \text{if } q_t = 0 \\ \left(\dfrac{\gamma - \delta\theta}{1-\delta}\right)\left(\dfrac{w_t}{\rho w_t - \kappa/\psi}\right) & \text{if } q_t > 0 \end{cases} \tag{9}$$

Parents who do not invest in the human capital of their children select some number of children. For parents who choose to invest, their demand for children depends on their wage income.[6] The higher the wage income is, the fewer the number of children they have; $\frac{\partial \eta_t}{\partial w_t} < 0$ and $\frac{\partial^2 \eta_t}{\partial w_t^2} > 0$.

For parents who decide to invest in children's human capital, the amount of quality investment for children is positively associated with their income. The parental child quality choice is

$$q_t = \begin{cases} 0 & \text{if } q_t = 0 \\ \dfrac{\delta\theta\rho}{(\gamma - \delta\theta)\kappa} w_t - \dfrac{\gamma}{(\gamma - \delta\theta)\psi} & \text{if } q_t > 0 \end{cases} \tag{10}$$

Regardless of human capital investment decision, parents save the fixed portion of their income. The amount of saving is

$$s_t = \frac{c_{t+1}}{1+r} = \frac{1-\gamma-\delta}{1-\delta} w_t \tag{11}$$

The next step is to clarify the factors that differentiate the parental choice on child quality. Since fertility and quality of children are chosen via utility maximization, parents' decisions are related to the indirect utilities derived from their choices. Substituting (9), (10), and (11) into the utility function yields the indirect utilities corresponding to child quality decisions. The indirect utilities

6) We assume that $\gamma > \delta\theta$. Parents who choose to invest in the human capital of their children also have some positive number of children.

of parents who do not invest in child quality is

$$V_{q_t=0}=\gamma\ln\frac{\gamma}{(1-\delta)\rho}+(1-\gamma-\delta)\ln\left(\frac{1-\gamma-\delta}{1-\delta}w_t\right) \tag{12a}$$

The indirect utilities of parents who provide a positive level of child quality is

$$V_{q_t>0}=\gamma\ln\left(\frac{\gamma-\delta\Theta}{1-\delta}\right)\left(\frac{w_t}{\rho w_t-\frac{\kappa}{\psi}}\right)+\delta\Theta\ln\left\{\frac{\delta\Theta\psi}{(\gamma-\delta\Theta)k}\left(\rho w_t-\frac{\kappa}{\psi}\right)\right\}$$
$$+(1-\gamma-\delta)\ln\left(\frac{1-\gamma-\delta}{1-\delta}w_t\right) \tag{12b}$$

Parents invest in child quality if and only if $V_{q_t>0}>V_{q_t=0}$. Therefore, the condition for human capital investment is

$$V^{*}=V_{q_t>0}-V_{q_t=0}>0 \tag{13}$$

Since V^{*} is a function of wage income, it is easy to see that a parental decision depends on their wage income level. It turns out that only parents whose income is above a threshold level invest in human capital for their children:

$$q_t>0 \;\Leftrightarrow\; w_t>\widetilde{w}=\frac{\gamma\kappa}{\delta\Theta\rho\psi} \tag{14}$$

Equation (14) suggests an interesting story about parental choice on their children's quality. The condition indicates that parental altruism is stratified by level of wage income. It presents a tragic situation in which rich parents are more altruistic and poor parents are less altruistic in terms of human capital investment for their children. This behavioral difference is not derived from some ad hoc assumption, but obtained from a process of utility maximization.

The threshold level of income in equation (14) is affected by parameters of utility function and human capital formation function. An increase in θ or ψ raises the rate of return to child quality and thus induces parents to invest in human capital at a lower wage. Likewise, a decrease in the unit costs of quality κ lowers the level of threshold wage income. An increase in ρ raises the opportunity costs of having a child and thus makes low income parents be more inclined to invest in child quality. Thus stratified parental altruism can at least be partially modified by economic policies targeted at the parameters of the model.

4. The Dynamic System

The stock of capital at time $t+1$ is determined by aggregate supply of savings at time t[7]

$$K_{t+1} = L_t s_t \tag{15}$$

The number of workers at time $t+1$ is

$$L_{t+1} = \eta_t L_t \tag{16}$$

From (15) and (16), and the definition of $k = K/L$, we have

$$k_{t+1} = \frac{s_t}{\eta_t} = \begin{cases} \dfrac{1-\gamma-\delta}{\gamma}\rho w_t & \text{if } w_t \in (0, \tilde{w}) \\ \dfrac{1-\gamma-\delta}{\gamma-\delta\theta}\left(\rho w_t - \dfrac{\kappa}{\psi}\right) & \text{if } w_t \in (\tilde{w}, \infty) \end{cases} \tag{17}$$

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7) In this overlapping generation model, old capital at time t wears out completely at time $t+1$.

where $\tilde{w}=\frac{\gamma\kappa}{\delta\Theta\rho\psi}$. As discussed above, the quality of workers at time $t+1$ depends on their parents' quality investment decision at time t. The stock of human capital at time $t+1$ is

$$h_{t+1}=\begin{cases} 1 & \text{if } w_t\in(0,\ \tilde{w}) \\ \left\{\frac{\delta\Theta\psi}{(\gamma-\delta\Theta)\kappa}(\rho w_t-\frac{\kappa}{\psi})\right\}^{\Theta} & \text{if } w_t\in(\tilde{w},\infty) \end{cases} \tag{18}$$

The wage income of workers at time $t+1$ is

$$w_{t+1}=A(1-\alpha)k_{t+1}^{\alpha}h_{t+1}^{1-\alpha}.$$

Combining (17) and (18) yields the dynamic path of the wage income

$$\begin{aligned} w_{t+1}&=\Phi(w_t) \\ &=\begin{cases} A(1-\alpha)(\frac{1-\gamma-\delta}{\gamma})^{\alpha}(\rho w_t)^{\alpha} & \text{if } {}_t\in(0,\ \tilde{w}) \\ A(1-\alpha)(\frac{1-\gamma-\delta}{\gamma-\delta\Theta})^{\alpha}\left\{\frac{\delta\Theta\psi}{(\gamma-\delta\Theta)\kappa}\right\}^{\beta}(\rho w_t-\frac{\kappa}{\psi})^{\alpha+\beta} & \text{if } w_t\in(\tilde{w},\infty) \end{cases} \end{aligned} \tag{19}$$

where $\beta=\Theta(1-\alpha)$. Note that $\Phi(w_t)$ is strictly increasing over each interval of $(0,\ \tilde{w})$ and $(\tilde{w},\infty)$:

$$\Phi'(w_t)=\begin{cases} A\alpha(1-\alpha)(\frac{1-\gamma-\delta}{\gamma})^{\alpha}\rho^{\alpha}w_t^{\alpha-1}>0 & \text{if } w_t\in(0,\ \tilde{w}) \\ A(\alpha+\beta)(1-\alpha)(\frac{1-\gamma-\delta}{\gamma-\delta\Theta})^{\alpha}\left\{\frac{\delta\Theta\psi}{(\gamma-\delta\Theta)\kappa}\right\}^{\beta}\rho(\rho w_t-\frac{\kappa}{\psi})^{\alpha+\beta-1}>0 & \text{if } w_t\in(\tilde{w},\infty) \end{cases} \tag{20}$$

Since $\alpha<1$ and $\alpha+\beta<1$, we have

$$\lim_{w_t\to\infty}\Phi'(w_t)=0 \tag{21}$$

and

$$\Phi''(w_t) < 0 \quad \forall w_t. \tag{22}$$

5. The Steady-State Equilibrium

As can be seen from (17), (18) and (19), the evolution of wage income $\{w_t\}_{t=1}^{\infty}$ is determined by the evolution of per capita capital stock $\{k_t\}_{t=1}^{\infty}$ and human capital $\{h_t\}_{t=0}^{\infty}$. A steady-state equilibrium is a stationary level of wage income $\overline{w}$, such that

$$\overline{w} = \Phi(\overline{w}) \tag{23}$$

The stationary fertility rate $\overline{n}$ and the stationary level of quality $\overline{q}$ correspond to $\overline{w}$.

Since $\Phi(w_t)$ is a continuous function of w_t, a steady-state equilibrium exists if $\Phi(0) \geq 0$ and there exists w_t such that $\Phi(w_t) < w_t$.[8] From (19), (20) and (21), we have $\Phi(0) = 0$, $\lim_{w_t \to \infty} \Phi'(w_t) = 0$, and the strict concavity of $\Phi(w_t)$ over $(\tilde{w}, \infty)$. Therefore, $\Phi(w_t) < w_t$ for some $w_t > \tilde{w}$, Thus, a steady-state equilibrium exists.

However, the steady-state equilibrium need not be unique. Given the strict monotonicity and strict concavity of $\Phi(w)$ over the interval $(\tilde{w}, \infty)$, and given that $\Phi(0) = 0$ and $\lim_{w_t \to \infty} \Phi'(w_t) = 0$, multiple nontrivial steady-state equilibria exist if $\Phi(\tilde{w}) < \tilde{w}$ and there exists a $w_t > \tilde{w}$ such that $\Phi(w_t) > w_t$. From (19), (20), (21) and (22), it is possible that the dynamic

8) We ignore a trivial degenerating steady state with $\overline{w} = 0$.

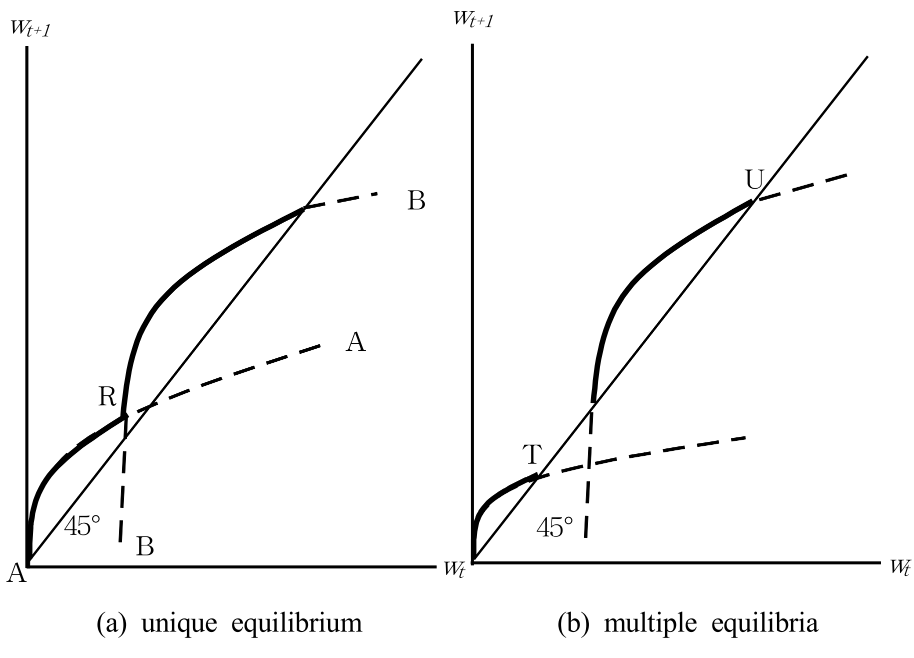
system is characterized by multiple steady-state equilibria for some range of parameter values.

Furthermore, the evolution of w_t may show a non-monotonic pattern. The slope of the dynamical system in the neighborhood to the right of $\widetilde{w}$ is steeper than that in the neighborhood to the left of it. In other words,

$$\lim_{w_t \to \widetilde{w}_-} \Phi'(w_t) < \lim_{w_t \to \widetilde{w}_+} \Phi'(w_t) \tag{24}$$

Figure 1 illustrates the evolution of wage income. There are two possible cases. Figure 1(a) characterizes the dynamic evolution of an economy that has a unique steady-state equilibrium and show a non-monotonic growth pattern. Starting along AA, the economy accelerates once it reaches a threshold income level at point R and gets on the path BB toward long-run economic growth.

〔Figure 1〕 Evolution of Wage Income



(a) unique equilibrium (b) multiple equilibria

The second case in figure 1(b) characterizes a dynamic system where there exist nontrivial multiple steady-state equilibria, T and U. One equilibrium corresponds to a lower income and higher fertility steady-state equilibrium(T) and the other to a higher income and lower fertility steady-state(U). An economy with low initial income converges to the steady-state below the threshold income level.

6. The Evolution of Wage, Fertility, and Human Capital

The dynamic path toward steady-state equilibrium demonstrates that parental altruism can play a unique role in explaining the established relation between fertility and growth. The pace of evolution of wage income may not be monotonic, as shown in figure 1(a). The pace declines as wage income grows towards $\tilde{w}$, accelerates once $\tilde{w}$ is passed, and declines once again as the economy approaches the steady-state equilibrium $\overline{w}$. Thus, as long as parents do not spend their income to improve the quality of their children $(w_t < \tilde{w})$, the rate of wage growth-and, thus, output-declines over time. The level of output remains low and fertility remains high. However, once the wage income becomes sufficiently high to induce parents to care about the quality of their children, the economy experiences accelerated growth that is accompanied by a declining fertility rate. Ultimately, growth slows and the economy converges to a higher output and lower fertility steady-state equilibrium.

An economy may face a poverty trap when its income dynamics are characterized by multiple steady-state equilibria, as shown in figure 1(b). If an economy starts from a low level of initial income, it may monotonically converge to a poverty trap of a high fertility and a low per capita capital stock with no human capital investment. Parental choice is constrained by a low income $(w_t > \tilde{w})$ and they do not invest in the human capital of their children. Without an exogenous shock or public policies designed to overcome poverty the economy cannot break out of the trap.

Ⅲ. Policy Analysis

The analysis in the previous section suggests the possibility of a vicious circle of poverty characterized by a low level of steady-state income. Poverty prevents the altruism of parents and thereby reproduces the poverty in the next generation. Fertility remains high, human capital is not accumulated, and wage income stays low permanently.

Expansion of these results to an economy in which both the poor and the rich live together requires additional assumptions. The most convenient assumption for this purpose is a perfectly separated dual economy. The economy consists of two sectors; low income sector and high income sector. Each sector has its own production function, which exercises no external effect on the other. We use this two-sector version of the previous model to examine the effects of policies targeted to the poor.[9)]

One can think of many policy options that may help a poor family escape a poverty trap. The strategies often suggested for this situation are income subsidies, subsidies to education and public provision of human capital. The effectiveness of each proposal is analyzed in this section.

1. Income Subsidy

Can income subsidy be viable policy to overcome poverty? This is an old issue that has been debated since the English Poor Law in the eighteenth century. Malthus(1798) argued in his Principle that the Poor Law offered little help for the plights of the English poor.

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9) For a complete extension of the model beyond this, see Lee(2005).

"The poor laws of England tend to depress the general condition of the poor in these two ways. Their first obvious tendency is to increase population without increasing the food for its support. ⋯ Secondly, the quantity of provision consumed in workhouses upon a part of the society that cannot in general be considered as the most valuable part diminishes the share that would otherwise belong to more industrious and more worthy members, and thus in the same manner forces more to become dependent."

Our model does not allow us to evaluate his second claim, but it permits us to appraise the merit of the first.

Suppose that the government initiates an income subsidy program to help poor families escape their miserable situation. Assume that the amount of b_0 is provided as a subsidy to poor parents in every generation. Then the parents' budget constraint becomes

$$\rho w_t n_t + \kappa q_t n_t + s_t \le w_t + b_0 \tag{7a}$$

Parents have as many children as

$$n_t^s = \begin{cases} \dfrac{\gamma(w_t + b_0)}{(1-\delta)\rho w_t} & \text{if } w_t \in (0, \widetilde{w}_s) \\ \left(\dfrac{\gamma - \delta\theta}{1-\delta}\right)\left(\dfrac{w_t + b_0}{\rho w_t - \dfrac{\kappa}{\psi}}\right) & \text{if } w_t \in (\widetilde{w}_s, \infty) \end{cases} \tag{9a}$$

where $\widetilde{w}_s = \widetilde{w}$. If parents choose to invest in the human capital of their children, they purchase quality

$$q_t^s = \frac{\delta\theta\rho}{(\gamma - \delta\theta)\kappa} w_t - \frac{\gamma}{(\gamma - \delta\theta)\psi} \quad \text{if } w_t \in (\widetilde{w}_s, \infty) \tag{10a}$$

Parents save a fixed proportion of their incomes, now including the subsidy. The amount of income parents save is

$$s_t^s = \frac{1-\gamma-\delta}{1-\delta}(w_t + b_0) \tag{11a}$$

Strikingly, the dynamic path of the wage income boils down to

$$w_{t+1}^s = \Phi_s(w_t)$$
$$= \begin{cases} A(1-\alpha)\left(\frac{1-\gamma-\delta}{\gamma}\right)^{\alpha}(\rho w_t)^{\alpha} & \text{if } w_t \in (0, \tilde{w}) \\ A(1-\alpha)\left(\frac{1-\gamma-\delta}{\gamma-\delta\theta}\right)^{\alpha}\left\{\frac{\delta\theta\psi}{(\gamma-\delta\theta)\kappa}\right\}^{\beta}\left(\rho w_t - \frac{\kappa}{\psi}\right)^{\alpha+\beta} & \text{if } w_t \in (\tilde{w}, \infty) \end{cases} \tag{19a}$$

which is exactly same as the evolution of wage income without income subsidy.

The reason the income subsidy does not work is simple. The poor parents will not be induced to spend income to raise the level of child quality. Therefore, workers from poor families continue to be unskilled in the next generation, $h_{t+1} = h_t = 1$. The poor parents receiving the income subsidy have more children $(n_t^s > n_t)$, which exhaust the subsidy as well as their increased savings$\left(\frac{1-\gamma-\delta}{1-\delta} b_0\right)$. As a result, per capita capital stock $k_{t+1} = \frac{s_t}{n_t}$ remains the same and the evolution of wage income is not affected at all. Figure 2(a) illustrates the evolution of wage income under income subsidy. In this model, income subsidy is not a practical option for escape from the poverty trap.

2. Income Subsidy Proportional to the Number of Children

The model will next be used to analyze an once popular social welfare program. Suppose that the government offers a subsidy of b_1 per each child parents have.

The budget constraint of the parents receiving this subsidy becomes

$$\rho w_t n_t + \kappa q_t n_t + s_t \leq w_t + b_1 n_t \tag{7b}$$

where $w_t + b_1 n_t < \tilde{w}$.

Since the opportunity cost of raising a child is lowered to $\rho w_t - b_1$, parents have more children

$$n_t^n = \begin{cases} \left(\frac{\gamma}{1-\delta}\right)\left(\frac{w_t}{\rho w_t - b_1}\right) & \text{if } w_t \in (0, \tilde{w}_n) \\ \left(\frac{\gamma - \delta\theta}{1-\delta}\right)\left(\frac{w_t}{\rho w_t - b_1 - \frac{\kappa}{\psi}}\right) & \text{if } w_t \in (\tilde{w}_n, \infty) \end{cases} \tag{9b}$$

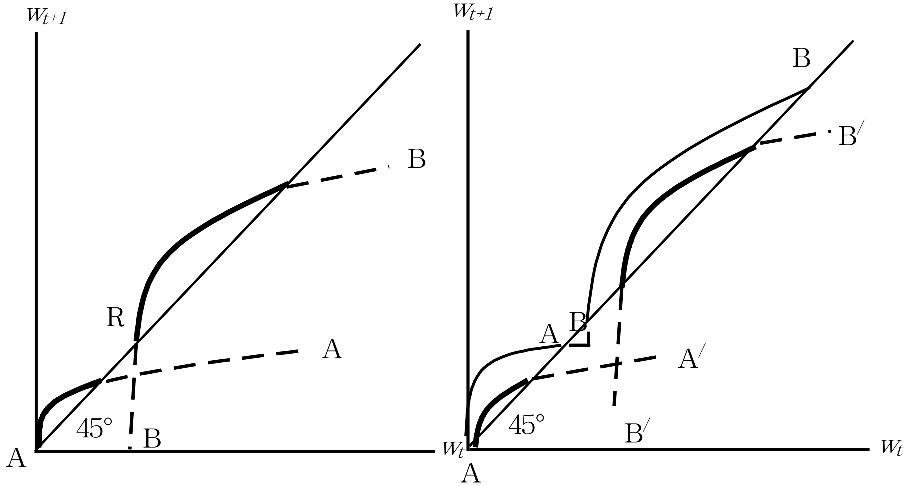
If parents choose to invest in the human capital of their children, they purchase quality as much as

$$q_t^n = \frac{\delta\theta\rho}{(\gamma - \delta\theta)\kappa}(\rho w_t - b_1) - \frac{\gamma}{(\gamma - \delta\theta)\psi} \qquad \text{if } w_t \in (\tilde{w}_n, \infty)$$

As before, parents save a fixed proportion of their income. The amount of income parents save is

$$s_t^n = \left(\frac{1-\gamma-\delta}{1-\delta}\right) w_t \tag{11b}$$

〔Figure 2〕 Effects of Income Subsidy



(a) income subsidy

(b) income subsidy proportional to number of children

The dynamic path of wage income is

$$w^n_{t+1} = \Phi_n(w_t)$$
$$= \begin{cases} A(1-\alpha)\left(\frac{1-\gamma-\delta}{\gamma}\right)^{\alpha}(\rho w_t - b_1)^{\alpha} & \text{if } w_t \in (0, \tilde{w}) \\ A(1-\alpha)\left(\frac{1-\gamma-\delta}{\gamma-\delta\Theta}\right)^{\alpha}\left\{\frac{\delta\Theta\psi}{(\gamma-\delta\Theta)\kappa}\right\}^{\beta}\left(\rho w_t - b_1 - \frac{\kappa}{\psi}\right)^{\alpha+\beta} & \text{if } w_t \in (\tilde{w}, \infty) \end{cases}$$

(19b)

Under this program, the steady-state wage income declines and the transition path becomes less steep. Therefore, this kind of income subsidy program aggravates rather than improving the plight of the poor. Furthermore, the poor family faces more difficulty in taking off because the threshold income level is higher under this subsidy

$$\widetilde{w}_n = \frac{\gamma\kappa}{\delta\Theta\rho\psi} + \frac{b_1}{\rho} > \widetilde{w} = \frac{\gamma\kappa}{\delta\Theta\rho\psi}.$$

Figure 2(b) illustrates the evolution of wage income under income subsidy proportional to the number of children. This kind of income subsidy moves the dynamic paths from AA to AA′ and BB to BB′, making transition to the higher steady-state more difficult. As Malthus argued, the obvious effect of the policy is to increase unskilled population without increasing wage income for its support.

3. Subsidy to Education

There are many programs that stimulate human capital accumulation of children. The most familiar is a subsidy to education. Suppose that the government subsidies a portion of education costs born by parents. The budget constraint of the parents becomes

$$\rho w_t \eta_t + (\kappa - e) q_t \eta_t + s_t \leq w_t \tag{7c}$$

where e is the amount of education subsidy per child.

Obviously, this program does not affect the decision of parents who choose not to invest in human capital of their children. If parents choose to invest in human capital of children, they purchase the quantity of education

$$q_t^e = \frac{\delta\Theta\rho}{(\gamma - \delta\Theta)(\kappa - e)} w_t - \frac{\gamma}{(\gamma - \delta\Theta)\psi} \quad \text{if } w_t \in (\widetilde{w}_e, \infty) \tag{10c}$$

The number of children per family becomes

$$\eta_t^e = \begin{cases} \dfrac{\gamma}{(1-\delta)\rho} & \text{if } w_t \in (0, \widetilde{w}_e) \\ \left(\dfrac{\gamma - \delta\Theta}{1-\delta}\right)\left(\dfrac{w_t}{\rho w_t - \dfrac{\kappa - e}{\psi}}\right) & \text{if } w_t \in (\widetilde{w}_e, \infty) \end{cases} \tag{9c}$$

The dynamic path of the wage income is

$$w_{t+1}^{e} = \Phi_{e}(w_t)$$

$$= \begin{cases} A(1-\alpha)\left(\frac{1-\gamma-\delta}{\gamma}\right)^{\alpha}(\rho w_t)^{\alpha} & \text{if } w_t \in (0, \widetilde{w}_e) \\ A(1-\alpha)\left(\frac{1-\gamma-\delta}{\gamma-\delta\theta}\right)^{\alpha}\left\{\frac{\delta\theta\psi}{(\gamma-\delta\theta)(\kappa-e)}\right\}^{\beta}\left(\rho w_t - \frac{\kappa-e}{\psi}\right)^{\alpha+\beta} & \text{if } w_t \in (\widetilde{w}_e, \infty) \end{cases} \tag{19c}$$

As shown in figure 3(a), a subsidy to education can change the income dynamics of poor families. By moving the phase line segment from BB to $B^{/}B^{/}$ the transition to the higher income become feasible. Since the cost of education is lowered by the amount of subsidy, parents have incentive to invest in the quality of their children at a lower income level. This is clear when we compare the threshold income levels for education investment.

$$\widetilde{w}_e = \frac{\gamma(\kappa-e)}{\delta\theta\rho\psi} < \widetilde{w} = \frac{\gamma\kappa}{\delta\theta\mu\psi} \tag{14c}$$

where $\widetilde{w}_e$ is the threshold income level under education subsidy. This policy helps on economy take off earlier. It is particularly effective when wisely targeted to families around the threshold level.

4. Public Provision of Human Capital

Suppose that the government designs a tax system to finance a public education or child care services. Parents are obliged to pay a tax the amount of which is proportional to the number of their children.

The budget constraint under the tax system is

$$\rho w_t n_t + \kappa q_t n_t + s_t \leq (1-\tau n_t) w_t \tag{7d}$$

where $\tau \in (0,1)$ is a per child tax rate. With the tax revenue the government purchases education goods from the market and distributes them to children. Or, government may be as efficient as private education sector in producing education goods. When parents do not invest in child quality, human capital in the unskilled grows only from education provided by the government:

$$h^{g}_{t+1} = (1+\psi\frac{\tau w_t}{\kappa})^{\theta} \qquad \text{if } w_t \in (0, \widetilde{w}_g) \tag{8d}$$

where $\frac{\tau w_t}{\kappa}$ is the quantity of education per child financed by the education tax.

The parents have fewer children under the tax

$$\eta^{e}_{t} = \begin{cases} \dfrac{\gamma}{(1-\delta)(\rho+\tau)} & \text{if } w_t \in (0, \widetilde{w}_g) \\ (\dfrac{\gamma-\delta\theta}{1-\delta})(\dfrac{w_t}{\rho w_t - \frac{\kappa}{\psi}}) & \text{if } w_t \in (\widetilde{w}_g, \infty) \end{cases} \tag{9d}$$

The amount of saving is the same as before

$$s^{g}_{t} = (\frac{1-\gamma-\delta}{1-\delta}) w_t \tag{11d}$$

The dynamic path of wage income is

$$w^{g}_{t+1} = \Phi_g(w_t)$$
$$= \begin{cases} A(1-\alpha)(\dfrac{1-\gamma-\delta}{\gamma})^{\alpha}\{(\rho+\tau)w_t\}^{\alpha}(1+\dfrac{\psi\tau}{\kappa}w_t)^{\beta} & \text{if } w_t \in (0, \widetilde{w}_g) \\ A(1-\alpha)(\dfrac{1-\gamma-\delta}{\gamma-\delta\theta})^{\alpha}\left\{\dfrac{\delta\theta\psi}{(\gamma-\delta\theta)\kappa}\right\}^{\beta}(\rho w_t - \dfrac{\kappa}{\psi})^{\alpha+\beta} & \text{if } w_t \in (\widetilde{w}_g, \infty) \end{cases}$$

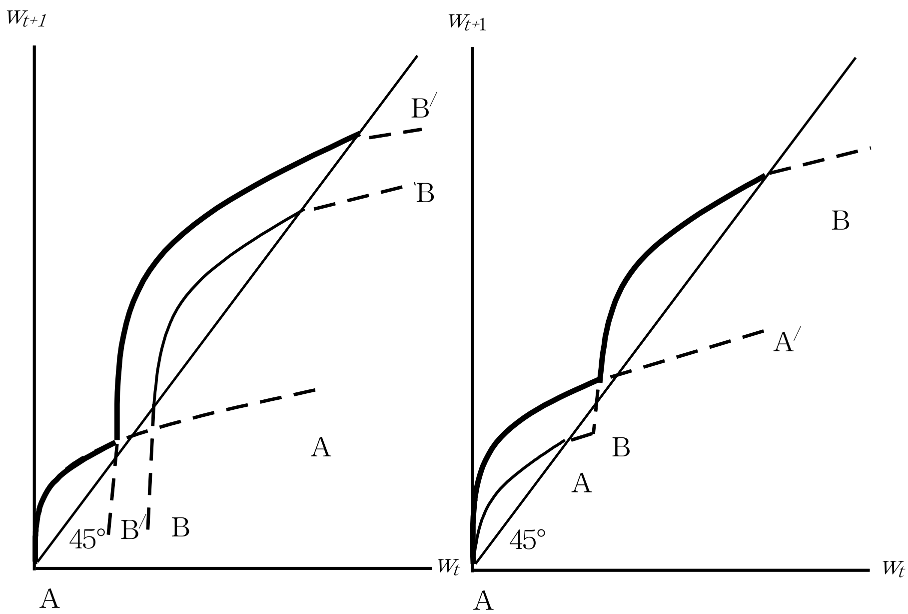
(19d)

where $\widetilde{w}_g = \dfrac{\gamma\kappa}{\psi(\delta\theta\gamma - (\gamma - \delta\theta)\tau)} > \widetilde{w} = \dfrac{\gamma\kappa}{\delta\theta\rho\psi}$. For $w_t \in (0, \widetilde{w}_g)$, since the slope is positive and steeper than that of $\Phi'(w_t)$ in equation (20)

$$\Phi'_e(w_t) = A(1-\alpha)\left(\frac{1-\gamma-\delta}{\gamma-\delta\theta}\right)^{\alpha}(\rho+\tau)\left(1+\frac{\psi\tau}{\kappa}w_t\right)^{\beta-1} w_t^{\alpha-1}\left\{\alpha+\frac{\psi\tau}{\kappa}(\alpha+\beta)w_t\right\} \tag{20d}$$

So the wage income grows faster below the threshold income level. For $w_t \in (\widetilde{w}_g, \infty)$, the dynamics of wage income are not affected by the education tax.

〔Figure 3〕 Effects of Education Subsidy and Public Provision of Human Capital



(a) subsidy to education (b) public provision of human capital

The evolution of wage under public provision of human capital is depicted in figure 3(b). By moving the phase line upward from AA to $AA^{/}$, this policy can facilitate the transition to the higher income steady-state.

This model sheds light on a much debated issue - whether public education (child care services) system can bring a higher rate of growth or higher level of income (Glomm and Ravikumar 1992, Benabou 1996, Durlauf 1996, Fernandes and Rogerson 1998). Public provision of human capital by government can alleviate poverty when low income induces parents not to invest in child quality. However, an education finance system makes no difference when parents voluntarily invest in human capital of their children. This conclusion follows from the assumption that public education is as efficient as private education.

Ⅳ. Conclusion

This paper develops a general equilibrium model that emphasizes the interactions among poverty, parental altruism and economic growth. The evolution of wage income is determined by two mechanisms. The first is the parents' investment in child quality. The more parents spend on their children's quality, the higher the wage income their children will obtain. The second is the feedback from fertility to per capita capital stock. Low fertility raises the per capita capital ratio and thus increases wage income.

The model in this paper establishes that fertility and child quality choices are stratified by income level. There is a threshold income level below which parents do not invest in human capital for their children. If poverty dominates parental altruism toward children, an economy may end up in a low income steady-state.

This stratified parental altruism suggests the possibility of a vicious circle of poverty. Poverty prevents the altruism of parents and thereby reproduces the poverty in the next generation. Income subsidies cannot solve this dynamic poverty problem. When a family is trapped in poverty, income subsidy policy has no effect on income, and sometimes aggravates poverty. However, the results of this paper are not as dismal as they look. Public policies such as subsidy to education and public provision of human capital can be viable options for overcoming poverty. These policies are not only effective but also self-sufficient.

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abstract

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이 인 재

본고는 세대간 중첩모형(overlapping generation model)을 이용하여 자녀의 인적자본 형성에 관한 부모의 이타적 동기를 분석함으로써 인적자본투자, 출산율 및 경제성장의 상호관계를 설명하는 새로운 메커니즘을 제시한다. 주요한 결론은 자녀의 인적자본 형성에 영향을 미치는 부모의 이타주의적 동기가 일정 수준 이상의 소득수준에서만 작동한다는 것이다. 즉, 임금소득이 일정 수준을 넘어야만 부모는 자녀의 인적자본 제고를 위한 투자를 시작하며, 그 결과 출산율이 하락하고 1인당 자본비율이 증가한다. 따라서 소득의 증가는 비단조적(non-monotonic) 형태를 취한다.

이러한 계층화된 부모의 이타적 동기는 다중균형(multiple equilibria)의 하나로서의 동태적 빈곤 함정(poverty trap)이 존재할 수 있음을 시사한다. 소득수준이 낮은 가구는 세대가 거듭함에 따라 높은 출산율, 낮은 인적자본과 낮은 소득으로 특징지어지는 균제 상태(steady state)로 수렴될 수 있다. 교육에 대한 보조와 공적 인적자본형성정책 등은 이러한 동태적 빈곤 함정을 벗어나게 하는 정책수단이다. 그러나 소득보조정책은 빈곤을 극복하는 효과적인 수단이 아님이 증명된다.

핵심용어 : 세대간 중첩모형, 다중균형, 인적자본투자, 이타적 동기, 출산율, 경제성장, 빈곤 함정.